Now suppose the second order term of the outer expansion is known. Then the normal velocity near the surface can be calculated from the Taylor series

$$[\rho_e v_e]_y = -\psi_x(x, y) = -\{y\psi_{1yx}(x, 0) + [1/(R)^{1/2}][\psi_{2x}(x, 0) + y\psi_{2yx}(x, 0)] + 0(1/R)\}$$

In particular, at $y = \Delta^*$ we find

$$[\rho_e v_e]_{y=\Delta^*} = [\rho_e v_e]_{y=0} - \Delta^*(d/dx)(\rho_e u_e) + 0(1/R)$$

which, together with (11), leads to Fannelop's result. Similarly, at $y = \delta$ we find

$$[\rho_e v_e]_{y=\delta} = [\rho_e v_e]_{y=0} - \delta(d/dx)(\rho_e u_e) + 0(1/R)$$

or

$$[v_e]_{y=\delta}/u_e = \rho_w v_w/\rho_e u_e + d\delta^*/dx - [(\delta - \delta^*)/\rho_e u_e](d/dx)(\rho_e u_e) \quad (12)$$

which is just the formula of Li and Gross.

From the preceding analysis, we have three possible boundary conditions, all equivalent, to apply to the second order term of the outer expansion:

1) The stream function or effective blowing velocity at the surface, given by (8), (10), or (11). This condition is the most straightforward to apply in weak interaction theory, and is the accepted version by most practitioners of the method of inner and outer expansions.

2) The stream function vanishing on an effective displacement surface, as suggested by Fannelop. This procedure has the drawback that the solution is needed for $y < \Delta^*$ to continue with the second order boundary-layer term, as evidenced by (5). If only the second order outer solution is needed, this approach may be advantageous.

3) The flow angle at the outer edge of the boundary layer, as given by Li and Gross. This procedure has the same drawback as item 2, but appears to be natural for strong interaction problems. In that case, of course, the method of inner and outer expansions is not applicable in the form developed here.

References

[1] Li, T. Y. and Gross, J. F., "Transverse curvature effects in axisymmetric hypersonic boundary layers," AIAA J. 2, 1868–1869 (1964).

[2] Fannelop, T. K., "Displacement thickness for boundary layers with surface mass transfer," AIAA J. 4, 1142–1144 (1966).

Reply to O. R. Burggraf

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1) Li and Gross[1] obtained an expression for v_e/u_e which in two-dimensional flow reduces to

$$v_e/u_e = \rho_w v_w/\rho_e u_e + d\delta^*/dx - [(\delta - \delta^*)/\rho_e u_e](d/dx) \times (\rho_e u_e) \quad (1)$$

(Similar expressions have been derived by Mann[2] and Thyson-Schurmann.[3]) I do not question the validity of Eq. (1). I have some doubts with regard to its general utility since it does not define v_e/u_e with precision, δ being arbitrary.

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The point made in my paper[4] is that, in the past, Eq. (1) has been interpreted erroneously as an expression for the effective body slope required in pressure-interaction theory. Li and Gross in Ref. 1 do not make their position clear in this regard, an omission that may have caused a good deal of confusion (cf. Sec. 2 below). The expression derived by Li and Gross [Eq. (1)] and that of Fannelop,[4] given below

$$v_e/u_e = \rho_w v_w/\rho_e u_e + d\delta^*/dx - [(\Delta^* - \delta^*)/\rho_e u_e](d/dx)(\rho_e u_e) \quad (2)$$

are related but not equivalent. Equation (1) provides the proper matching condition for the inviscid flow at $y = \delta$; Eq. (2) gives the corresponding condition at $y = \Delta^*$, i.e., the effective body slope. The matching conditions can be related by

$$(v_e/u_e)_{y=\delta} = (v_e/u_e)_{y=\Delta^*} + (\delta - \Delta^*)(\partial/\partial y)(v_e/u_e) + 0(1/R) \quad (3)$$

where

$$(\delta - \Delta^*)\frac{\partial/\partial y}{v_e/u_e} = -\frac{\delta - \Delta^*}{\rho_e u_e}\frac{d}{dx}(\rho_e u_e) + 0\left(\frac{1}{R}\right)$$

2) For hypersonic applications Li and Gross suggest the approximation $\delta \approx \delta^*$. All terms involving the arbitrary quantity δ then cancel, and (v_e/u_e) can be evaluated easily. In a recent paper Lewis et al.[5] have considered a numerical example for which all the terms in the Li and Gross transverse-curvature formula are examined. The authors make the following statement, "We note that the effect of the terms neglected by Li and Gross under the hypersonic approximation caused a 40% and 50% change in $\tan^{-1}(d\delta^*/dx)$ at $M_\infty = 5.64$ and 10, respectively." At moderate hypersonic Mach numbers at least it appears that a formula that does not require arbitrary approximations would be useful.

3) If the proper matching condition for the perturbed inviscid flow is known at $y = y_1(x)$, the corresponding matching condition at $y = y_2(x)$ (within the boundary-layer region) is obtained simply by accounting for the change in the inviscid flow over a distance $y_2 - y_1 \sim 0(^1/R^{1/2})$, i.e.

$$(v_e/u_e)_{y=y_2} = (v_e/u_e)_{y=y_1} + (y_2 - y_1)(\partial/\partial y)(v_e/u_e) + 0(1/R) \quad (4)$$

In view of this simple relation, the lengthy rederivation undertaken by Burggraf[6] in order to obtain the matching conditions at $y = 0$ and $y = \delta$ seems rather unnecessary. We note that from continuity

$$(y_2 - y_1)(\partial/\partial y)(v_e/u_e) = -[(y_2 - y_1)/\rho_e u_e](d/dx)(\rho_e u_e) + 0(1/R)$$

Thus, from Eqs (2) and (4) with $y_2 = 0$, $y_1 = \Delta^*$ we obtain

$$(v_e/u_e)_{y=0} = \rho_w v_w/\rho_e u_e + d\delta^*/dx + (\delta^*/\rho_e u_e)(d/dx)(\rho_e u_e)$$

which is Burggraf's Eq. (11).

4) The definition of an effective displacement thickness Δ^* offered by Fannelop[4] has two distinct advantages: i) It is in accord with the classical theory since in the absence of mass transfer it reduces to the displacement thickness δ^*; ii) It is capable of precise definition.

The alternate forms suggested by Burggraf reduce in the absence of mass transfer to the following:

(a) $(v_e/u_e)_{y=0} = (d\delta^*/dx) + (\delta^*/\rho_e u_e)(d/dx)(\rho_e u_e)$

(b) $(v_e/u_e)_{y=\delta^*} = (d\delta^*/dx)$

(c) $(v_e/u_e)_{y=\delta} = d\delta^*/dx - [(\delta - \delta^*)/\rho_e u_e](d/dx)(\rho_e u_e)$

The inherent simplicity of the second alternative, which is the one preferred in boundary-layer theory, is striking. Any advantages that (a) and (c) may possess have yet to be demonstrated.

References

[1] Li, T. Y. and Gross, J. F., "Transverse curvature effects in axisymmetric hypersonic boundary layers," AIAA J. **2**, 1868–1869 (1964).

[2] Mann, W. M., Jr., "Effective displacement thickness for boundary layers with surface mass transfer," AIAA J. **1**, 1181–1182 (1963).

[3] Thyson, N. A. and Schurmann, E. E. H., "Blowing effects on pressure interaction associated with cones," AIAA J. **1**, 2179–2180 (1963).

[4] Fannelop, T. K., "Displacement thickness for boundary layers with surface mass transfer," AIAA J. **4**, 1142–1144 (1966).

[5] Lewis, C. H., Marchand, E. O., and Little, H. R., "Mass transfer and first-order boundary layer effects on sharp cone drag," AIAA Paper 66-33 (1966).

[6] Burggraf, O. R., "The choice of boundary conditions in viscous interaction theory," AIAA J. **4**, 1145–1146 (1966).

Step-Induced Boundary-Layer Separation Phenomena

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In Ref. 1, Halprin presents a three-region hypothesis describing the flow upstream of a cylindrical step immersed in supersonic flow. The hypothesis was developed from flow visualization studies and pressure data obtained from the plate on which the step was mounted. A particular mention was made that the "boundary layer separates from the plate and becomes a viscous shear layer which impinges upon the front face of the cylindrical step," whereas for a two-dimensional (forward facing) step, the shear layer passes over the step without impinging on its face. The contrast in behavior is used to explain the difference obtained in pressure on the plate ahead of the two types of steps, this being a primary concern of the paper.

The validity of the flow model presented in Fig. 2 of Ref. 1 may be enhanced by results of studies of the pressure distribution on the cylinder itself. Sykes[2] presents such data obtained at $M = 1.96$; his pertinent results are shown in Fig. 1. The pressure coefficient along the stagnation line of the cylinder, the shock location, and the outline of the separated boundary layer are presented in terms of the cylinder diameter, which was 0.307 in. It may be seen that the minimum pressure point on the cylinder does not occur at the cylinder-plate junction, but rather at a small distance out along the cylinder. The minimum coincides approximately with the lower edge of the separated boundary layer. Thus the pressures on the cylinder support Halprin's hypothesis of a "third region" of low-energy flow below the separated boundary layer. The rise in pressure at the plate-cylinder junction similarly agrees with his suggestion that the junction is a stagnation region.

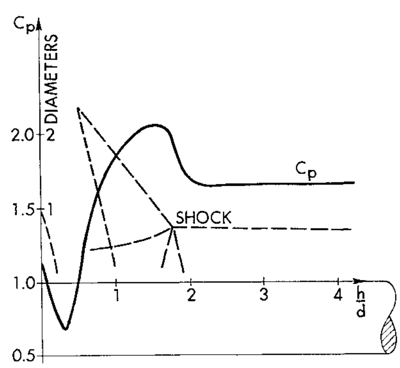

Fig. 1 Shock shape and stagnation-line pressure distribution from Ref. 2.

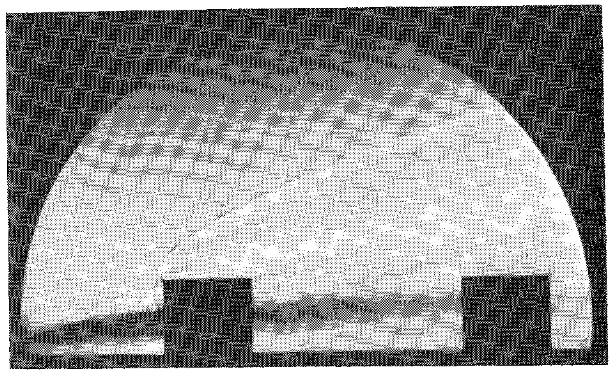

Fig. 2 Schlieren photograph of 0.6-in. diam, 0.5-in. high cylinders.

The present writer is investigating the drag of cylinders of various heights mounted on a flat plate. Tests have been made with single and multiple cylinders, the latter arranged to simulate various fastener patterns.[3] Figure 2 is a schlieren photograph of two identical cylinders of 0.6-in. diam and 0.5-in. height, oriented in line with the $M = 5$ flow. (The aft cylinder has been found to have a negligible effect on the flow ahead of the forward unit, which is the region of interest here.) The separated boundary layer is clearly evident, as is the low-energy flow region below it. The flow is essentially identical to the model proposed by Halprin.

There is unquestionably a model size effect on the flow around cylinders; Fig. 3 shows that, for small heights, the boundary layer does not separate. This absence of separation is also evident in pressure measurement made on short cylinders. The influence of model diameter is seen by comparing Figs. 2 and 4, for which the heights are identical.

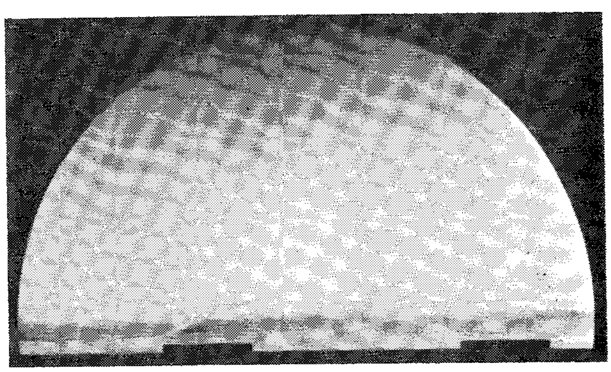

Fig. 3 Schlieren photograph of 0.6-in. diam, 0.063-in. high cylinders.

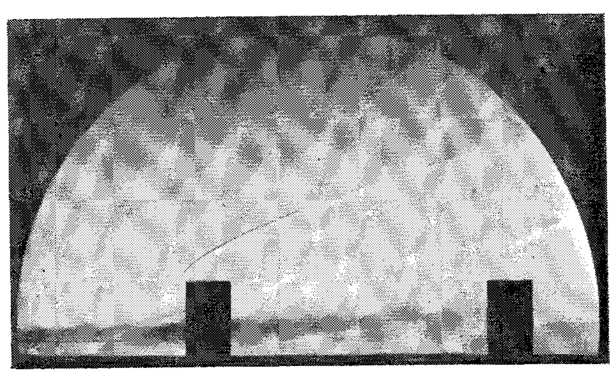

Fig. 4 Schlieren photograph of 0.3-in. diam, 0.5-in. high cylinders.

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